Mean Reversion Strategies

Introduction to Quantitative Trading

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Objectives

- Introduction to quantitative finance.
- Example of a quantitative trading strategy.
- Application of mathematical concepts to finance.

Outline

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Quantitative Trading

- Quantitative trading is a collection of trading strategies that uses quantifiable variables to inform trading decisions.
- It can be high or low frequency, and can be applied to all assets provided there is reliable numerical information available.

Other Forms of Trading

- Macro-Trading: Analysis of macroeconomic factors.
- Technical Analysis: Study of past price movements to capture market psychology.

High Frequency Trading

- Order of magnitude below a few seconds. Most often down to ms.
- Grew in the mid 00s, became most common in the 10s.
- Highly automated asset classes: equities and foreign exchange.

High Frequency Trading

Implications of high frequency trading.

- Overall more efficient markets, in particular on optimal execution.
- At microstructure level: new properties appear. Active field of research.
- Algo versus algo interactions, which can be complex to understand.
  Flash crash on S&P500 of May 6th, 2010.

Price Trends

The price of an asset as a function of time is not purely random.

- Both macro trading and quantitative trading attempt to monetise such trends.
- We often differentiate factors which are either *endogenous* versus *exogenous* to the price processes.

Market Microstructure: Random Walk

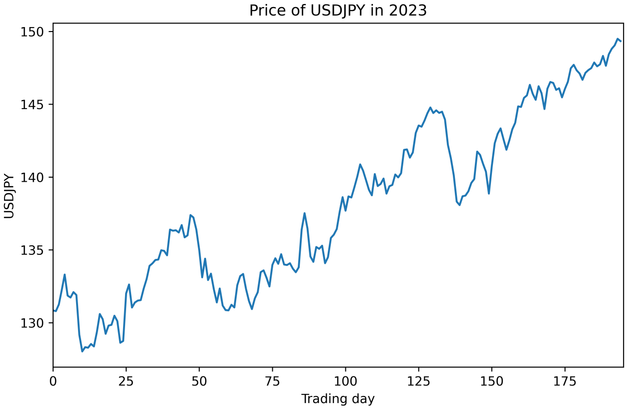

Figure 1: Example of Trending Price Process

Price Trends

- When identifying those trends, we define the concept of *reversion to the mean*.
- The quantitative strategies that exploit this property are called *mean reversion strategies*.
- When used across multiple assets, we call those *StatArb*.

Outline

Elements of Market Microstructure

Definition (Return)

$$r(t) = \frac{m(t)}{m(t-1)}$$

A more compact and common representation is the log return.

$$\begin{aligned}\log r(t) &= \log \frac{m(t)}{m(t-1)} \\ &= \log\left(1 + \varepsilon_m(t)\right) \\ &\overset{\text{Taylor}}{\simeq} \varepsilon_m(t)\end{aligned}$$

Market Microstructure: Random Walk

- The dynamics of the mid price $(m(t_n))_{n\in\mathbb{N}}$ are often represented as a random walk.
- Random walks are processes with random **steps**.
- The most commonly used is the **Gaussian Random Walk**.

Market Microstructure: Random Walk

Definition (Gaussian Random Walk)

Let $(X_i)_{i\in\mathbb{N}}$ an independent identically distributed variables such that $X_i \hookrightarrow \mathcal{N}(0,1)$. We call Gaussian Random Walk the process $(W_i)_{i\in\mathbb{N}}$ such that.

$$W_0 = 0$$
$$W_k = W_{k-1} + X_k, \forall k \in \mathbb{N}^*$$

Market Microstructure: Random Walk

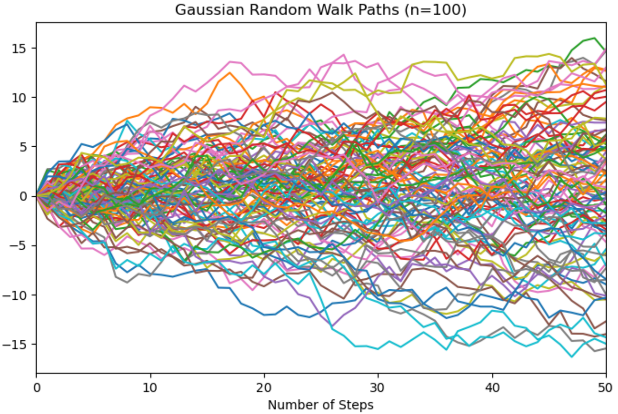

Figure 2: Example of Gaussian Random Walk Simulation

Market Microstructure: Random Walk

An example of continuous martingale is the Wiener Process.

Definition (Wiener Process)

The Wiener Process $W(t)$ is a stochastic process such that.

1. **Initialisation** $W(0) = 0$
2. **Independent Increments** $\forall t > 0, \delta \geq 0, s < t$ the increments $W(t+\delta) - W(t)$ are independent of $W(s)$
3. **Gaussian Increments** $\forall t > 0, \delta \geq 0$, $(W(t+\delta) - W(t)) \hookrightarrow \mathcal{N}(0, \delta)$
4. **Continuity** $W(.) \in \mathcal{C}^0(\mathbb{R})$

Asset Return Decomposition

Random walks do not have any trend, by construction. We need a model that captures price trends.

Definition (Asset Return Decomposition)

We can write the return of an asset $r_i(t)$ as a function of the market return $r_m(t)$.

$$r_i(t) = \alpha_i(t) + \beta_i \times r_m(t) + \varepsilon(t)$$

with β_i the market risk and ε a random process.

GBM

We can't always express directly the dependencies between the price process and another known time series.

Definition (Intuitive Model)

We introduce a drift $\mu \in \mathbf{R}$ and W_t a Wiener process such that the price of the asset is as follows.

$$S(t) = \mu \times t + \sigma \times W(t)$$

with $\sigma > 0$.

GBM

However, the most standard representation is the GBM, using (stochastic) differential equations.

Definition (Geometric Brownian Motion)

We introduce a drift $\mu \in \mathbf{R}$, a constant volatility $\sigma \geq 0$, and a Wiener process W_t.

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

This model is used in Black-Scholes for example. The GBM is a diffusion process.

OU Process

Another popular stochastic model, used for example for interest rates is the OU process.

Definition (Ornstein–Uhlenbeck Process)

We define the stochastic differential equation of the Ornstein–Uhlenbeck process.

$$dr_t = \theta(\mu - r_t)dt + \sigma dW_t$$

with $\theta, \sigma > 0$ and $\mu \in \mathbf{R}$.

It is well studied and has closed form conditional expectation and variance. OU is a mean-reversion process, with μ the long term price and θ the speed of reversion.

Outline

Fitting

- Once we have identified a potential mathematical model to describes a price, we will compute its parameters to represent the data we possess.
- This is called the *fitting* of a process.

Fitting: Intuitive Model

We observe the impact of the different parameters.

$$S(t) = \mu \times t + \sigma \times W(t)$$

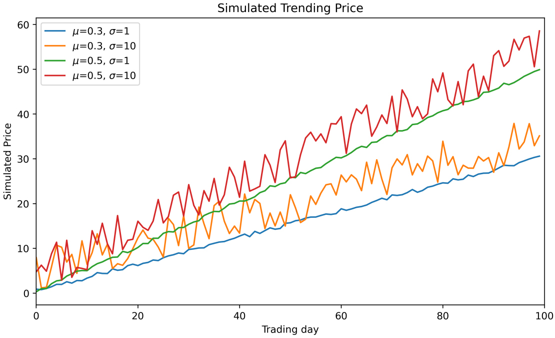

Figure 3: Simulation for different values of μ and σ

Fitting: GBM

Given the differential equation of the GBM

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

An intuitive estimator is the following...

$$S(t+1) - S(t) = \mu S(t) \times (t+1-t) + \sigma S(t) \times (W(t+1) - W(t))$$

$$S(t+1) = S(t) \times (1 + \mu + \sigma\varepsilon(t))$$

with $\varepsilon \hookrightarrow \mathcal{N}(0,1)$

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s[k+1] = s[k] * (1 + mu + sigma * random())
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Fitting: GBM

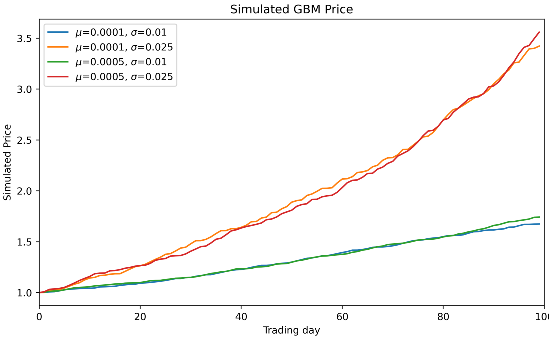

Figure 4: Simulation for different values of μ and σ

...But it's actually not quite correct!

Fitting: GBM

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

We can prove however with more rigorous math that there is a more accurate way to simulate, by solving the equation (Ito's formula).

$$S(t) = S(0) \times exp\left(\left(\mu - \frac{\sigma}{2}\right)^2 t + \sigma W_t\right)$$

We still need a little bit of math to properly simulate the models. :-)

Fitting: GBM

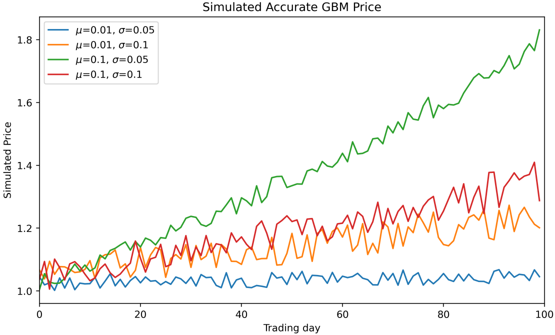

Figure 5: Simulation for different values of μ and σ

Testing

- Once we've found candidate parameters, we need to simulate the outcome on historical data.
- That process is called *backtesting* and is used to compute an error metric against our historical data.
- The most common error metric is called the Sum of Squared Residuals (SSR).

We will iterate that process until we have a good enough fit.

Application: Back to Trading

- When our model is good enough, we can identify the real trends from the local noise.
- Under the mean reversion assumption, if the price diverts too much from the trend, it will revert back.
- When we use multiple assets to take opposite positions (risk-neutral), this becomes *StatArb*.

Market Microstructure: Random Walk

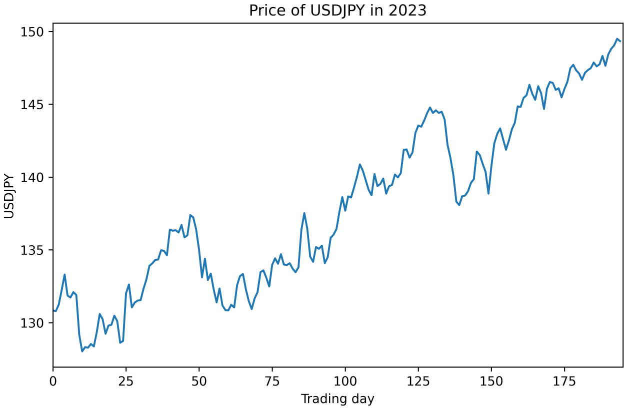

Figure 6: Example of Trending Price Process

Conclusion

- Quantitative trading is the field of trading that relies on **mathematics and data** to formulate trading strategies. It can be used at **high frequency**.
- One of such strategies is called **mean reversion** and tries to capture trends in prices. **Mathematical models** are used to represent the price process and are fitted using historical data.
- Based on the model prediction, the trader or algorithm can decide to take a position and **monetise** the reversion to the mean.

Thank you! :-)